Quantum Thermalization and the Expansion of Atomic Clouds

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I. Expansion of an Atomic Cloud: Classical or Quantum?

Motivation: In the cold atomic ‘time-of-flight’ measurements, one suddenly releases the confining potential and the atomic cloud expands. Typically, it has been assumed that this expansion is governed by a purely classical Newtonian or wave kinematics. Naively, it is not at all obvious why this works. After all, before releasing the trapping potential, one may be in a quantum regime with Bose condensation or Fermi-degeneracy. How can these atoms suddenly behave like classical cannon balls?

Main question: Are there instances where the quantum time evolution of a macroscopic system is qualitatively different from the equivalent classical system?

II. Expansion into the vacuum

The expansion of an atomic cloud into vacuum is the same when computed fully quantum-mechanically or classically!

III. Expansion into a cold bath

We start with a hot subsystem A in a cold bath. How will A thermalize?

Classical thermalization is diffusive, \( \partial_t T = D \nabla^2 T \) so \( \Delta T \sim t^{-d/2} \).

Relativistic fermions thermalize instantaneously.

Nonrelativistic fermions thermalize ballistically, \( \Delta E \sim t^{-d} \).

But bosons show a crossover from ballistic behavior, at high bath temperatures, to diffusive at low bath temperatures!

IV. Method of Modular Hamiltonian

Separate a system into two parts \( \text{A} \) and a bath \( \text{B} \).

The initial density matrix is a product of a thermal state in A and a thermal state in B.

\[
\rho_0 = \frac{1}{Z_A Z_B} e^{-\beta_A \mathcal{H}_A} \otimes e^{-\beta_B \mathcal{H}_B}
\]

It’s easier to compute the modular Hamiltonian: \( \mathcal{M} = -\log \rho \)

For noninteracting particles, the time evolution is simply:

\[
\mathcal{M}(t) = \sum_{kk'} m_{kk'} e^{-\beta \left( E_k - E_{k'} \right)} c_d^k c_{d'}^{k'} + \log Z
\]

The modular matrix decays as a ballistic powerlaw,

\[
\Delta m_{kk'}(t \gg 1) = 2 \Delta \delta(0) \left( \frac{L_A - 1}{2L_B} \right) \sim \frac{1}{t^{d/2}}
\]

though the Greens function (and the energy) can have different time-dependence, since \( G(t) = \left[ \epsilon_1(t) - \eta \right]^{-1} \).

V. Experimental realization

1. Trap an atom cloud
2. ‘Build a wall’ between A and B
3. Heat up system A
4. At \( t=0 \), remove wall
5. At later time, build wall
6. Remove all atoms in B
7. Measure kinetic energy in A using Time-Of-Flight